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The recovery of liquid from flowing foams

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Abstract

This paper examines the recovery of liquid from stable overflowing foams. The foams are formed by bubbling gas into the bottom of a column and the liquid is collected from the foam that flows over the lip at the top. The paper demonstrates, using a foam drainage equation, that the recovery of liquid will rapidly decrease towards an asymptotic value as the foam height is increased. An expression for this liquid recovery is then developed. That the liquid recovery becomes constant as foam height is increased in a stable foam is also demonstrated experimentally. The mathematical analysis of the problem suggests that the amount of liquid collected is proportional to the gas rate squared. This relationship is verified experimentally for an aqueous foam.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Flowing columns of foam are found in many industrial applications, including foam fractionation, column froth flotation and foam stripping columns. One of the key parameters in any of these processes is the rate at which water moves up the column and overflows the top of the column. In froth flotation, for instance, the amount of undesired material that is collected is usually directly proportional to the amount of water collected. This makes the calculation of this recovery crucial to the prediction of the performance of this system. A second type of system in which the foam will display a very similar behaviour is if the foam is pumped around a pipe network which contains a vertical section followed by a horizontal pipe section. This paper attempts to predict the recovery, or overflow rate, of liquid.

In this paper the behaviour of a steady-state stable flowing column of foam will be examined. A stable foam has a surfactant concentration high enough to prevent either bursting at the top surface or internal coalescence. Later in the paper, the restriction of bursting at the top surface will be relaxed.

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2. The mathematical model

The mathematical model that will be used to describe the behaviour of the liquid in, and overflowing the column is based on the model developed by Leonard and Lemlich (1965) and independently rediscovered and used by Verbist *et al* (1996). The model describes a force balance between gravity, capillary suction and viscous drag in the Plateau borders. If flow is assumed to be in only the vertical direction, the velocity of the liquid in the Plateau borders of the foam can be expressed as follows (positive direction is upwards):

$$v_l = -k_1 A - \frac{k_2}{\sqrt{A}} \frac{\mathrm{d}A}{\mathrm{d}y} + v_g. \tag{1}$$

The inclusion of the gas velocity makes this equation the one-dimensional version of the two-dimensional model developed for flowing foams by Neethling *et al* (2000). In equation (1), v_l and v_g are the velocities of the liquid and gas respectively, A is the cross-sectional area of the Plateau border at a given height y and k_1 and k_2 are combinations of physical parameters:

$$k_1 = \frac{\rho g}{3 C_{PB} \mu} \tag{2}$$

$$k_2 = \frac{\left(\sqrt{\sqrt{3} - \frac{\pi}{2}}\right)\gamma}{6 C_{PB} \mu}. \tag{3}$$

In equations (2) and (3) ρ, γ and μ are the liquid density, surface tension and viscosity respectively. C_{PB} is the Plateau border drag coefficient. The drag coefficient has a value of about 50 when the liquid–gas interfaces of the Plateau border are immobile, with the value decreasing as the mobility increases.

More complex models that include the effects of viscous losses in the vertices as well as in the Plateau borders (Koehler *et al* 1999, Neethling *et al* 2002) can also be used, but their complexity makes the examination of the basic phenomena occurring within a flowing column more difficult.

2.1. Continuity in a flowing column

In this study, the overflowing foam column problem will be treated as one dimensional. Figure 1 illustrates the physical situation.

If it is assumed that the column of foam is at steady state, then continuity in the foam can be expressed by the following equation:

$$\frac{\mathrm{d}(v_l A \lambda)}{\mathrm{d}y} = -Q_{Remove}. \tag{4}$$

In equation (4), λ is the length of Plateau border per volume of foam, and is proportional to (bubble diameter)$^{-2}$. λ also allows for the calculation of the liquid content of the foam ($\varepsilon = A\lambda$). Q_{Remove} is the volumetric rate of removal of liquid from the foam per volume of foam. Below the overflow lip, the rate of liquid removal is, of course, zero, as the net flow through the column at any height is the same.

The system must be considered in two parts; the large proportion of the foam below the overflow lip, flowing upwards, and the smaller portion of the foam above the lip, which overflows and carries liquid with it.

2.2. Foam below the overflow lip

In the portion of the foam below the lip, there is no liquid removal or addition other than through the lower and upper boundaries, and there is a single liquid flowrate Q_l.

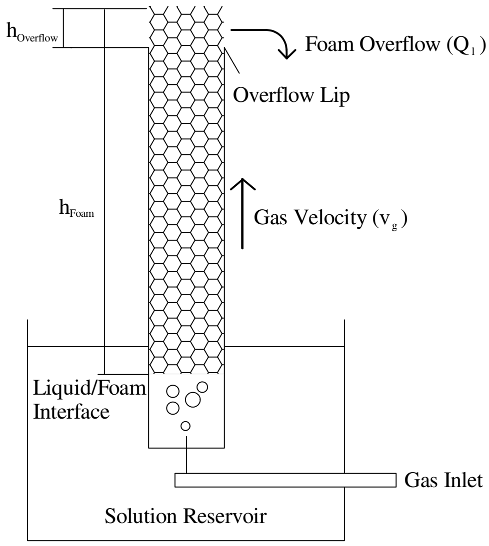

Figure 1. Schematic diagram of the flowing foam column.

In this part of the foam, the superficial liquid velocity can be expressed as follows:

$$\frac{Q_l}{A_{Column}} = \lambda\left(-k_1A^2 - k_2\sqrt{A}\,\frac{\mathrm{d}A}{\mathrm{d}y} + v_gA\right). \tag{5}$$

This can then be rearranged:

$$\frac{\mathrm{d}A}{\mathrm{d}y} = \frac{-\lambda k_1A^2 + \lambda v_gA - \frac{Q_l}{A_{Column}}}{\lambda k_2\sqrt{A}}. \tag{6}$$

To solve for the Plateau border area, A, with height, a boundary value for A at the liquid–foam interface is required. The following relationship holds:

$$A_{Interface} = \frac{\varepsilon_{Interface}}{\lambda}. \tag{7}$$

$\varepsilon_{Interface}$ is the liquid fraction of the foam at the foam–liquid interface. If the foam is assumed to be close packed and mono-dispersed, as in this work, then $\varepsilon_{Interface} \approx 0.26$. If it is assumed that the bubble packing at the interface is random and mono-dispersed, then $\varepsilon_{Interface} \approx 0.36$.

The liquid flowrate is a boundary condition which must be consistent with the result for overflowing portion of the foam. This can be determined by iteration.

2.3. *Foam above the overflow lip*

The portion of the foam above the overflow lip is more complex to describe, since the liquid flowrate is not constant, as liquid is removed as the foam flows over the lip. It will be assumed that the liquid flows over the lip at the same horizontal velocity as the gas, and can thus be expressed as follows:

$$Q_{Removal} = \frac{v_{g(out)}l_{lip}A\lambda}{A_{Column}}. \tag{8}$$

In equation (8), l_{lip} is the column circumference and $v_{g(out)}$ is the horizontal velocity of gas over the lip.

While any arbitrary horizontal liquid flow profile can be assumed, it will here be assumed that the horizontal gas velocity over the lip is constant with height. If the height of the foam above the lip is $h_{Overflow}$ (figure 1), then, since none of the bubbles burst at the top surface, the horizontal gas velocity is given by

$$v_{g(out)} l_{lip} h_{Overflow} = v_{g(in)} A_{Column}. \tag{9}$$

Thus equation (8) becomes

$$Q_{Removal} = \frac{v_{g(in)} A \lambda}{h_{Overflow}}. \tag{10}$$

The vertical gas velocity at any height above the overflow lip is given by

$$v_g = v_{g(in)} \frac{h_{foam} - y}{h_{Overflow}}. \tag{11}$$

In equation (11), y is the distance above the liquid–foam interface and h_{foam} is the total height of the foam. Equation (11) allows equation (1) to be re-written as

$$v_l = -k_1 A - \frac{k_2}{\sqrt{A}} \frac{\mathrm{d}A}{\mathrm{d}y} + v_{g(in)} \frac{h_{foam} - y}{h_{Overflow}}. \tag{12}$$

Substituting equations (10) and (12) into the continuity equation (4) and expanding, while assuming that bubble size, and thus λ, remains constant with height,

$$\frac{\mathrm{d}^2 A}{\mathrm{d}y^2} = \frac{-2k_1 A \frac{\mathrm{d}A}{\mathrm{d}y} - \frac{k_2}{2\sqrt{A}} (\frac{\mathrm{d}A}{\mathrm{d}y})^2 + v_{g(in)} \frac{h_{foam}-y}{h_{Overflow}} \frac{\mathrm{d}A}{\mathrm{d}y}}{k_2 \sqrt{A}}. \tag{13}$$

Equation (13) describes the Plateau border area, and hence the liquid fraction, between the overflow lip and the top of the foam. Both the value and gradient of A at the inlet to this region are required; these are obtained by integration of equation (6).

The final boundary condition required to fully describe the system is the top surface of the foam. Since no liquid or gas flows through the top surface, equation (1) yields the following condition:

$$A_{Top} = -\left(\frac{k_2}{k_1} \left(\frac{\mathrm{d}A}{\mathrm{d}y}\right)_{Top}\right)^{2/3}. \tag{14}$$

2.4. Solution method

A suitable solution strategy is to iterate on the liquid flow rate in the foam below the overflow lip (equation (6)), which gives the Plateau border area and gradient at the height of the overflow lip. Solving equation (13) gives values for A and its gradient at the top surface. This is repeated until the boundary condition (equation (14)) is met, and the overflow liquid rate becomes known.

3. Typical results from the model

In this section the properties of a typical numerical solution for a foam with a constant bubble size and overflowing the lip of a cylinder will be discussed, using the physical conditions given in table 1.

The physical constants chosen here are typical of foam systems previously studied (Neethling *et al* 2002). The choice of the liquid fraction at the interface being that of close packed spheres was arbitrary and does not significantly influence the results shown, except for very shallow foams. The drag coefficient of 30 is approximately that obtained by doing forced

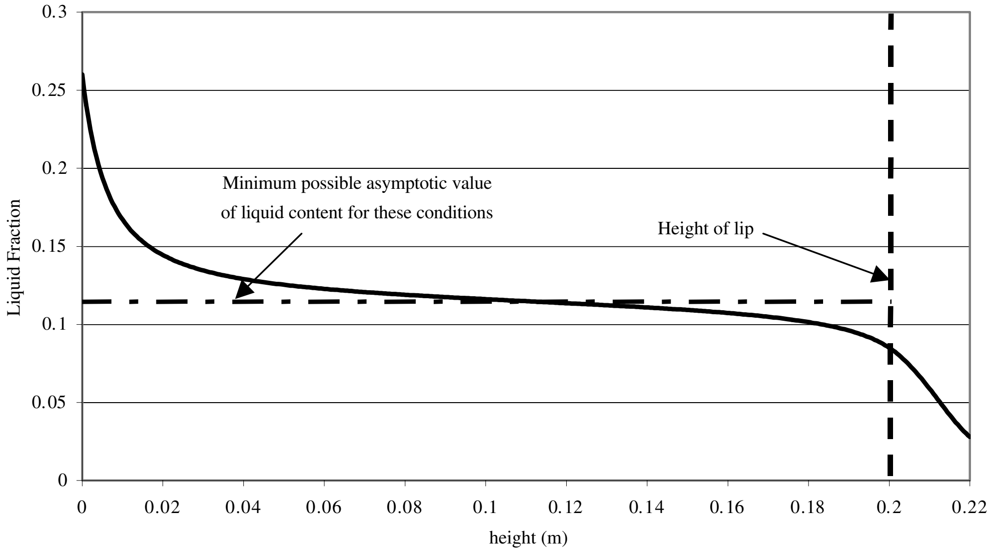

Figure 2. A typical profile of liquid fraction with foam height for a stable, overflowing foam as obtained from the numerical solution of the model.

Table 1. Typical operating conditions.

Column diameter, d_{column} (cm)	2.5
Foam height from interface to top surface, h_{foam} (cm)	22
Foam height above overflow lip, $h_{overflow}$ (cm)	2
Bubble diameter, d_b (mm)	1.5
Volumetric gas flowrate, Q_{gas} (ml min^{-1})	250
Plateau border drag coefficient, C_{PB}	30
Liquid density, ρ (kg m^{-3})	1000
Liquid viscosity, μ (Pa s)	0.001
Surface tension, γ (N m^{-1})	0.0528
Liquid fraction at interface, $\varepsilon_{interface}$	0.26

drainage experiments of the type carried out by, for instance, Verbist *et al* (1996), using the same surfactant system and bubble size as is presented here. This drag coefficient is less than the value of about 50 that would be found if the liquid gas interfaces in the Plateau borders were immobile, indicating that there is a certain amount of surface mobility in this system.

Figure 2 shows the variation in liquid fraction, ε, with foam height predicted by the model. This trend is typically observed for a very wide range of conditions, and will be discussed generically.

What is immediately evident from figure 2 is that, in the foam below the overflow lip, the sign of the second derivative of liquid fraction (and, for invariant bubble size, A) with respect to foam height changes from being positive in the lower portion of the foam to being negative towards the overflow lip. This is in contrast to the non-overflowing, steady state foams, such as the equilibrium liquid fraction profile and the steady forced drainage of liquid (Verbist *et al* 1996), in which both the first and second derivatives asymptote to zero as height increases and the liquid fraction and A thus approach a constant value. The second derivative also remains positive in those cases. It is possible to get such an asymptotic solution from equation (6); however, these do not occur when the region above the lip is included, as will be explained in the following sections.

3.1. Minimum liquid content asymptote

Consider a hypothetical, constant bubble size foam experiment in which the upward flowrates of liquid and gas can be manipulated independently. In this situation, as the foam height becomes infinite, for any given gas rate, v_g, there exists a range of values for Q_l, the upward volumetric flowrate of liquid for which the liquid fraction, ε (and thus Plateau border area, A), asymptotes to a constant value. This value is obtained from equation (5) by setting the gradient of A to zero, since it is the asymptotic value that is required:

$$A_{Asymptote} = \frac{v_g + \sqrt{v_g^2 - \frac{4k_1 Q_l}{\lambda A_{Column}}}}{2k_1}. \tag{15}$$

Equation (15) appears counter-intuitive in that it implies that the drier the foam, the greater the liquid flowrate upwards, Q_l. This is a result of the capillary suction disappearing as the asymptote is approached, combined with the fact that the downward effect of gravity is proportional to A^2, whereas the effect of the upward motion of the gas is only proportional to A. Equation (15) indicates that there is an upper limit to the upward liquid flowrate for which an asymptotic solution exists. This corresponds to the maximum liquid flowrate (and minimum liquid content) at which the viscous drag is able to balance gravity without a contribution from capillary suction. This upper flowrate is given by

$$Q_{l(Min.\,Asymptote)} = \frac{v_g^2 \lambda A_{Column}}{4k_1}. \tag{16}$$

The corresponding asymptotic value of A is

$$A_{Min.\,Asymptote} = \frac{v_g}{2k_1}. \tag{17}$$

It must be remembered that the maximum flowrate asymptote corresponds to the minimum liquid content asymptote.

Figure 2 also shows the liquid fraction corresponding to this asymptotic value of A. It is clear that, for the complete foam, the value of A will drop below the minimum possible asymptotic value and that Q_l will be higher than the maximum for that asymptote. Q_l is able to exceed this value because the capillary suction remains appreciable.

The lower value of A (or ε), and the higher flow rate Q_l than the asymptotic values, is a result of the boundary condition at the top of the foam. If at the level of the lip $\mathrm{d}A/\mathrm{d}y$ is small, then $\mathrm{d}^2A/\mathrm{d}y^2$ will also be small (from equation (13)). If $\mathrm{d}^2A/\mathrm{d}y^2$ is small, then $\mathrm{d}A/\mathrm{d}y$ will not vary much over the top region, which means that $\mathrm{d}A/\mathrm{d}y$ will remain small. For equation (14) (the top boundary condition) to be satisfied, $\mathrm{d}A/\mathrm{d}y$ must already be reasonably negative at the level of the lip ($\mathrm{d}A/\mathrm{d}y$ must already be of the same order of magnitude as required to satisfy the top boundary condition at the level of the lip). This condition cannot be satisfied for values of Q_l that result in a liquid content that asymptotes to a near constant value.

Figure 3 shows examples of hypothetical liquid fraction–foam height profiles that are obtained if Q_l can be varied independently of v_g. The profiles show two distinct limits of behaviour: the asymptotic behaviour described by equations (16) and (17), and values of Q_l at which the foam is completely dry at the lip level. For the full solution that includes the foam above the lip, Q_l cannot be varied independently of v_g and the range of Q_l values in which the valid solution exists must fall between the minimum asymptotic value and the limit at which the foam becomes dry at the lip. This is a remarkably narrow range: for this set of conditions, less than 1%, and, as will be shown in greater detail in the following section, the maximum asymptotic value of Q_l will generally closely approximate the full, iterative solution. The sensitivity of the solution to conditions below the lip explains why the estimated liquid flow

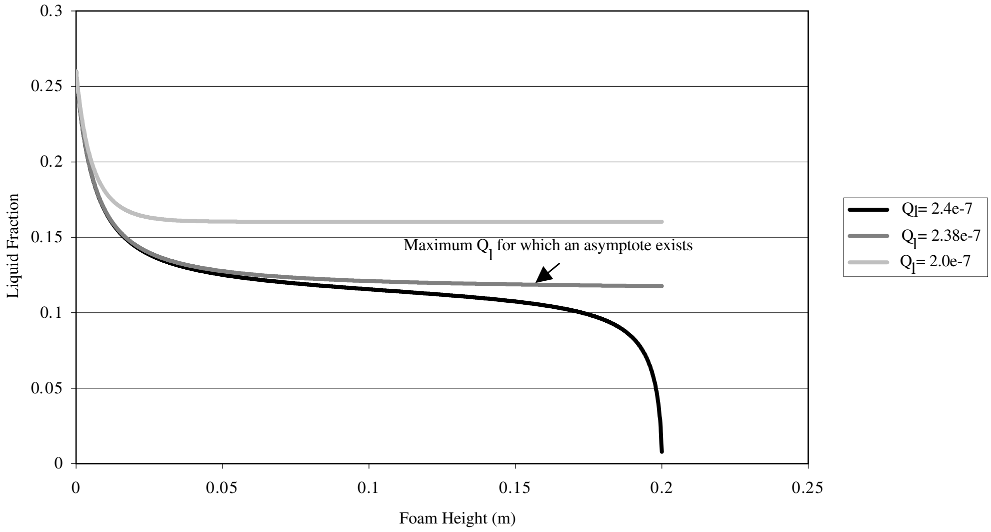

Figure 3. Liquid profiles in the region below the lip if it were possible to vary Q_l independently of v_g. Note that the difference between the last asymptotic liquid flow and the flowrate at which the foam is totally dry at the top is very small.

rate out of the column is virtually independent of the foam height or gas flow profile above the lip.

4. Solutions for the complete overflowing foam column

4.1. The effect of foam height on liquid recovery

One of the key operational variables in a flowing foam column is the foam height. Figure 4 shows the relationship between foam height and liquid recovery as obtained from a full numerical solution of the model, for two foam heights above the lip. The foam height above the lip only has an effect on the liquid recovery when the liquid fraction varies significantly with height. At greater foam heights, both curves asymptote to the value of Q_l given by equation (16).

It should be noted that, in general, experimental foam columns do not show a constant water recovery with foam height. This is because the average bubble size usually increases with height, due to coarsening or coalescence, resulting in a decrease in water recovery as the foam height increases. The effect of even small variations in bubble size on liquid recovery can be significant, as will be shown in the following section.

4.2. The effect of bubble size on water recovery

A second variable that affects the liquid recovery is the bubble size. The full model was solved using the conditions from table 1, but with varying bubble size, in order to calculate the liquid overflow rate. Figure 5 shows the liquid flowrate from the overflowing foam column when the bubble size in the foam is varied from 0.5 to 5 mm. Figure 5 also shows the liquid flowrate predicted by the asymptotic solution (equation (16)). Figure 5 shows that the effect of the bubble size on water recovery is far more significant than the effect of foam height.

Figure 5 further shows that the maximum asymptotic value for the liquid flowrate provides a very good estimate for the water recovery above a certain critical bubble size. Below this

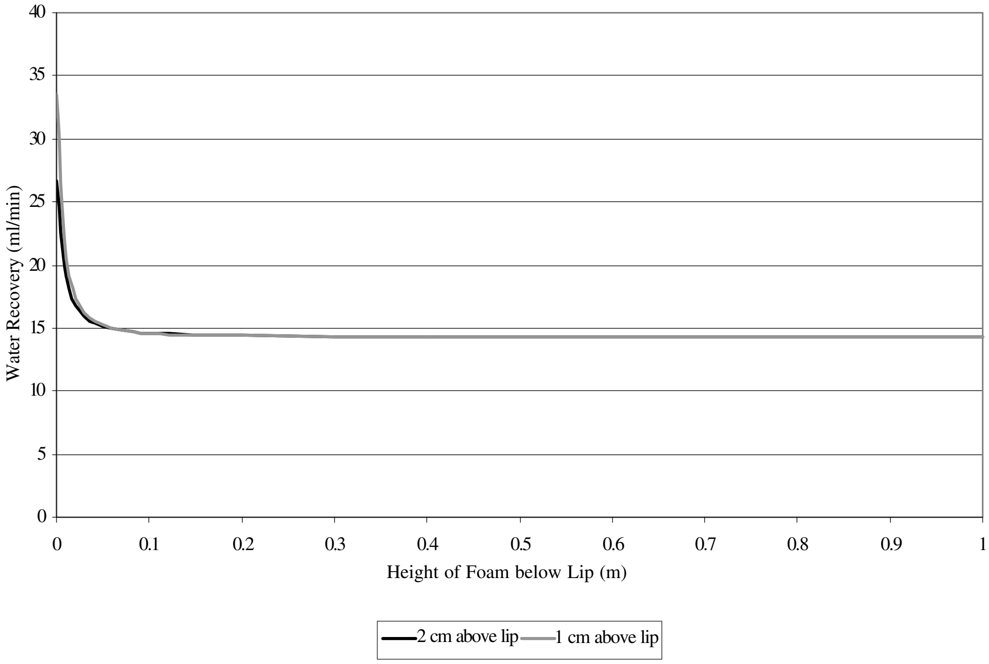

Figure 4. Water recovery from a stable foam as a function of foam height. Two different foam heights above the overflow lip are shown.

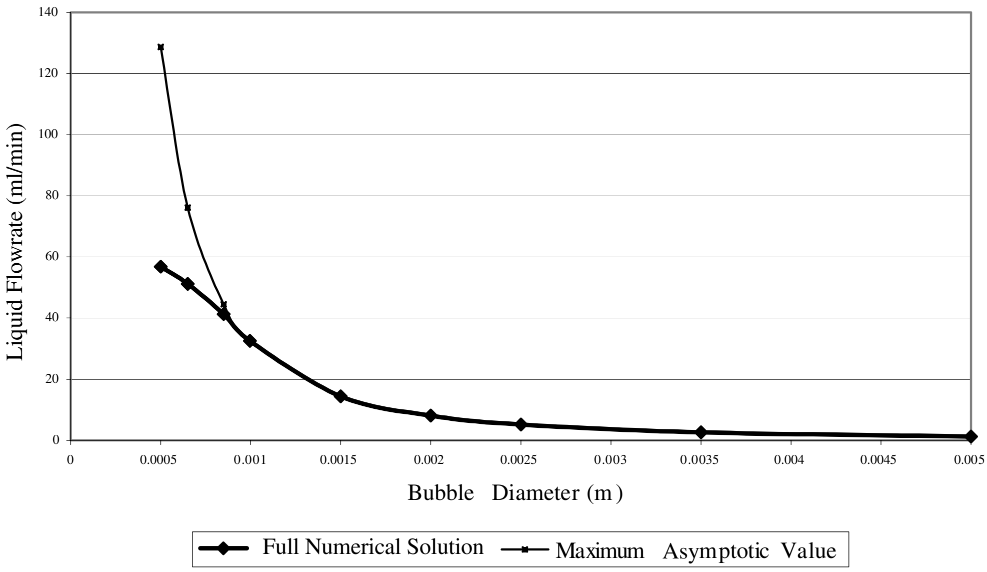

Figure 5. Water recovery from a stable foam as a function of bubble size.

critical bubble size, however, the maximum asymptotic value and the results of the numerical solution of the full model diverge.

The reason for this deviation is as a result of the value of A at the liquid–foam interface. For A to approach the asymptote, it must have a value greater than the asymptotic value given by equation (15) at the interface. As the bubble size decreases, the value of A at the liquid foam interface also decreases, while the minimum asymptotic value of A is unaffected. The critical bubble size at which A at the liquid foam interface is less than the minimum asymptotic value can be obtained from equation (17), and a suitable geometric relationship between λ and the bubble diameter. For these calculations, the foam is assumed to be a mono-dispersed Kelvin foam, which implies that all the bubbles are tetrakaidecahedra and $\lambda = 1.71/d_b^2$. If the foam were poly-dispersed, the proportionality in the relationship between λ and d_b would change and would be dependent on the degree of poly-dispersity, but the rest of the equations developed in this paper remain unaffected.

$$\lambda_{Critical} = \frac{2k_1\varepsilon_{Interface}}{v_g}. \tag{18}$$

For the conditions in table 1 equation (18) yields a critical bubble diameter of 0.995 mm, which is in excellent agreement with the point in figure 5 where the asymptotic and the full solutions diverge.

For bubble sizes smaller than the critical size, A starts below the minimum asymptote. In this case, equation (6) indicates that there exists a value of Q_l for which the gradient of A at the interface can be zero or small. If the gradient is negative at the interface, it will become more negative higher in the foam, and at an increasing rate. If, however, the gradient is small at the interface, the Plateau border area will not change significantly with height, up to the point at which A decreases rapidly to match the boundary condition at the overflow lip. This indicates that for bubble sizes less than the critical value, the entire foam will have essentially the same value of ε as at the liquid–foam interface, except for a small region below the top surface.

A simple expression exists for the recovery of liquid when the bubble size is below the critical size determined from equation (18):

$$Q_l = \varepsilon_{Interface} A_{Column}\left(v_g - \frac{k_1\varepsilon_{Interface}}{\lambda}\right). \tag{19}$$

Equation (19) is obtained by ignoring the gradient term in equation (5) and using A at the liquid–foam interface. This equation has near perfect agreement with the full solution for bubble sizes below the critical size.

It should be noted at this point that equations (18) and (19) are based on models for foam that have a low liquid fraction ($\lesssim$4%). The general argument, however, remains valid.

4.3. The behaviour of unstable foams

The focus of this paper thus far has been the behaviour of totally stable foams that do not have coarsening of the bubble size in the foam, or bursting on the upper surface. In this section, the bursting limitation will be relaxed, but the bubbles in the foam will still be considered not to coalesce or grow by diffusion.

The fraction of air entering the foam that overflows the lip as unburst bubbles will be referred to as α. Equation (6) remains the governing equation for foam below the overflow lip; however, the foam above the lip and the top boundary condition require modification. Equation (8) for the rate of removal of liquid remains valid, though equations (9) and (10) must be modified to include α:

$$v_{g(out)} l_{lip} h_{Overflow} = v_{g(in)} A_{Column}\alpha \tag{20}$$

$$v_g = v_{g(in)}\left(\frac{h_{foam} - y}{h_{Overflow}}\alpha + (1-\alpha)\right). \tag{21}$$

Substituting and expanding results in the governing equation for the foam above the overflow lip,

$$\frac{\mathrm{d}^2 A}{\mathrm{d}y^2} = \frac{-2k_1 A \frac{\mathrm{d}A}{\mathrm{d}y} - \frac{k_2}{2\sqrt{A}}(\frac{\mathrm{d}A}{\mathrm{d}y})^2 + v_{g(in)}\left(\frac{h_{foam}-y}{h_{Overflow}}\alpha + (1-\alpha)\right)\frac{\mathrm{d}A}{\mathrm{d}y}}{k_2\sqrt{A}}. \tag{22}$$

Equation (22) reverts to (13) when α is unity.

The boundary condition is obtained by applying equation (1) at the upper surface of the foam, and noting that the velocity of the liquid through the top surface is zero, but that the gas velocity depends on α. This yields

$$\left(\frac{\mathrm{d}A}{\mathrm{d}y}\right)_{Top} = -\frac{k_1}{k_2}A_{Top}^{\frac{2}{3}} + \frac{v_{g(in)}}{k_2}(1-\alpha)A_{Top}^{\frac{1}{2}}. \tag{23}$$

4.4. Asymptotic relationships

Figure 4 showed that the asymptotic overflow liquid flowrate is rapidly approached as the foam height increases, and when there is no surface bursting of the foam. As the bursting rate increases (α decreases), the column height at which the liquid flowrate asymptotes becomes larger.

There are two operating regimes, depending on whether the liquid content asymptotes to a constant value or not: first, the regime in which the foam surface boundary condition (equation (23)) is satisfied when the gradient of A at the surface is appreciable, and second, the regime in which the boundary condition is satisfied when the gradient is arbitrarily small. In the first regime, the value of A at the upper surface is less than the minimum asymptotic value; in the second it is not.

The second regime is described by setting the gradient of A to zero in equation (23), and limiting A to be greater than the minimum asymptotic value:

$$\frac{k_1}{k_2}A_{Top}^{\frac{3}{2}} = \frac{V_{g(Inlet)}}{k_2}(1-\alpha)A_{Top}^{\frac{1}{2}} \qquad \text{and} \tag{24}$$

$$\text{(from equation (15))} \qquad A \geqslant \frac{v_g}{2k_1}. \tag{25}$$

Solving simultaneously shows that the second regime occurs when $\alpha = \frac{1}{2}$. The asymptotic liquid flowrate that is approached as the foam height increases is the following in the two regimes:

$$\text{if } \alpha < \tfrac{1}{2}: \qquad Q_l = \frac{A_{Column}v_g^2\lambda}{k_1}(1-\alpha)\alpha \tag{26}$$

$$\text{if } \alpha \geqslant \tfrac{1}{2}: \qquad Q_l = \frac{A_{Column}v_g^2\lambda}{4k_1}. \tag{27}$$

Equation (26) is a combination of equations (24), (20) and (8), noting that if equation (23) is satisfied with a negligible gradient of A, then A is virtually constant over the entire region of the overflow. Equation (27) is identical to (16).

The first thing to note is that, for a sufficiently deep foam, the liquid recovery is independent of the bursting rate if α is greater than $\frac{1}{2}$. This means that more than half the bubbles must burst at the top surface before there is an appreciable change in the water recovery from a column. Since bursting and coalescence are often coupled, though, a system in which more than half the bubbles burst at the top surface will often violate the assumption that the bubble size does not vary with height. If the bubble size only changes slowly with height, then equations (26) and (27) could still be used with caution. The bubble size used to calculate λ must be the bubble size at the lip if $\alpha > \frac{1}{2}$, else it must be the average size flowing over the lip.

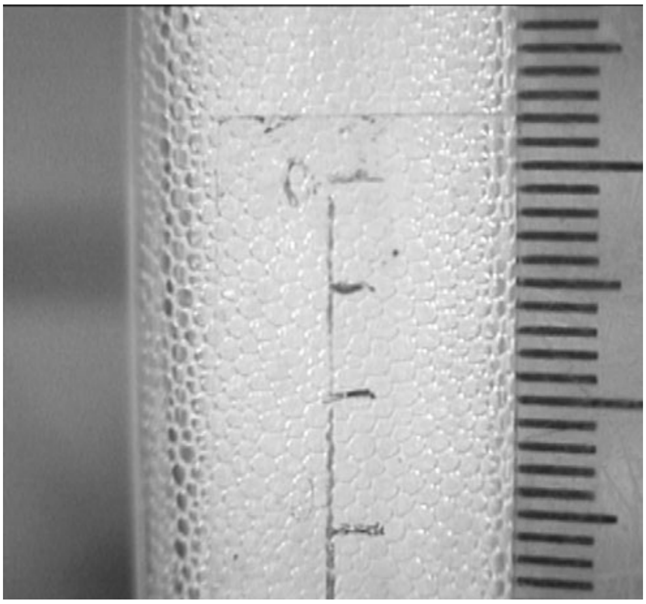

Figure 6. Image of the flowing bubbles in the foam column (divisions on the scale on the right-hand side of the column are 1 mm).

5. Experimental investigation

In order to investigate the practical applicability of the equations developed, an experimental investigation was carried out. This involved varying the air rate into the foam, as well as the height of the foam. The bubble size was not varied because of the difficulties in obtaining a range of mono-dispersed bubble sizes.

These experiments were carried out using a column of circular cross-section with a 1.4 cm internal diameter. The bubbles were produced using a glass frit that yielded a bubble size of about 1 mm. The exact size is hard to obtain, though it did not vary noticeably in the range of air rates used in these experiments. The column consisted of slotted together lengths of tube that allowed for the foam height to be varied. The reservoir of solution was large (about 20 l) and the foam was recycled back into it so fractionation of surfactant between the bulk and the foam is not a problem. Figure 6 shows an image of the flowing foam taken through the side of the column, which gives an indication of the bubble size, as well as demonstrating the mono-dispersed nature of the bubbles.

The column used a regulated and measured air supply. The foam overflows the top of the column and re-circulates back into the solution reservoir until steady state is reached (figure 1). The entire overflow of the column is collected for a fixed period to determine the overflow liquid rate.

It could be observed visually that the foam overflowing at the top surface was appreciably drier than the foam seen through the clear walls of the column. Except for the region near the liquid/foam interface and near the overflow lip, no appreciable variation in liquid content with height was observed. This is in line with the model prediction (see figure 2). Within the column the foam was totally stable, with no coalescence observed. The highly mono-dispersed nature of the bubbles, coupled with the comparatively small residence times in the column, meant that no appreciable amount of diffusion driven coarsening was observed, with no visible

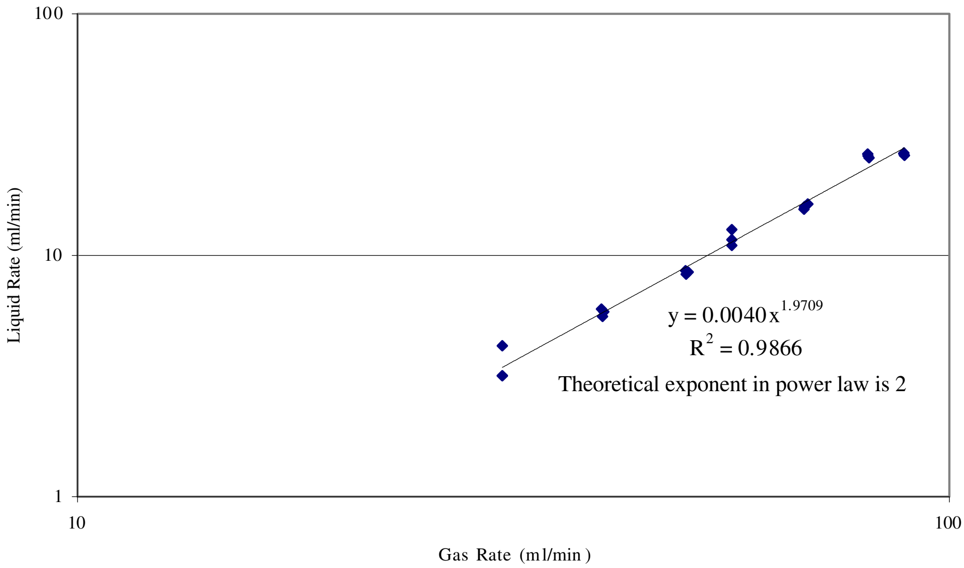

Figure 7. Experimental relationship between the gas flow into the column and the water recovery plotted on a log–log axis.

difference in bubble size, as seen through the walls, between the bottom and top of the column. This means that the experiment fulfilled the assumption of a stable foam.

At the top surface, though, a small amount of bursting was observed, possibly due to the low liquid content coupled with evaporation and dust from the environment. The amount of bursting was very small, though, and substantially less than the 50% that would result in equation (27) becoming invalid.

5.1. Air rate

Equation (27) (and (16)) predicts a power law relationship between liquid flow rate and gas flowrate, with an exponent of two. When these two values are plotted against one another (figure 7), the fit to a power law is excellent and the value of the exponent obtained from the least squares fit is very close to two (1.97). In these experiments the foam height was 80 cm.

A similar power law relationship is seen in the work of Verbist *et al* (1996) for the forced drainage of liquid through a foam. In that work it was found that the volumetric flowrate of the liquid relative to the foam ($Q_{l,Rel}$) was proportional to the velocity of the wavefront squared. The wavefront velocity, in turn, is equal to the velocity of the liquid relative to the foam ($v_{l,Rel}$). The full expression is as follows:

$$Q_{l,Rel} = \frac{v_{l,Rel}^2 \lambda A_{Column}}{k_1}. \tag{28}$$

This equation also holds in this work if the foam is sufficiently deep that the liquid content of most of the foam is near that of the asymptote. This equation, while similar to equations (16) and (27), is not identical. The main difference is that in the rest of this work, including equations (16) and (27), the liquid velocity quoted is not the velocity of the liquid relative to the foam, but rather the velocity of the liquid relative to the stationary column:

$$v_{l,Rel} = v_g - v_l. \tag{29}$$

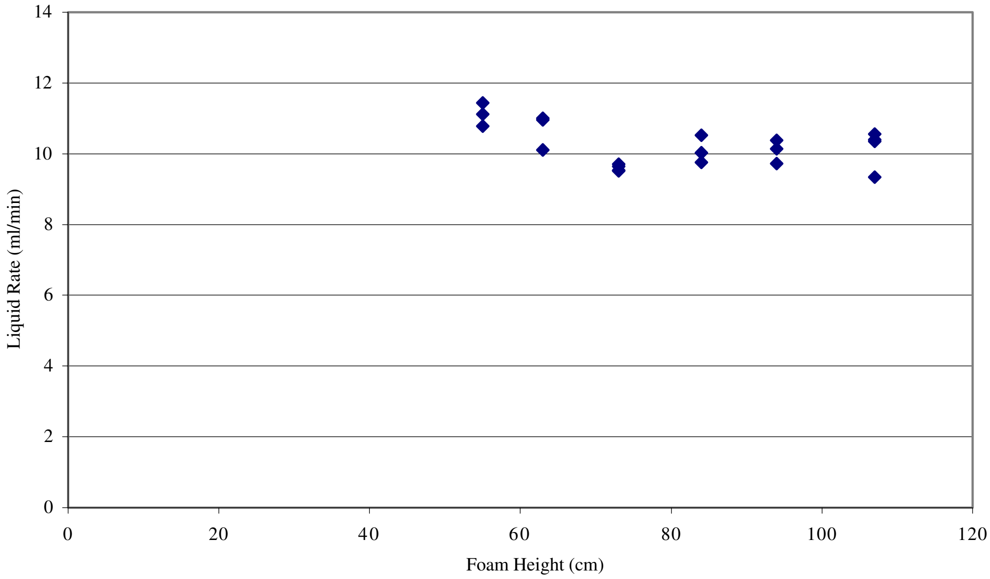

Figure 8. Experimental relationship between foam height and water recovery at a constant air rate.

Equation (28) can thus be written as follows:

$$(v_g - v_l)A\lambda = \frac{(v_g - v_l)^2 \lambda A_{Column}}{k_1}. \tag{30}$$

In order for both equations (30) and (27) to be valid, the velocity of the liquid relative to the column (in the region where it is asymptotic) and the velocity of the gas relative to the column must be proportional to one another. Combining equations (7), (16) and (17) gives the following result:

$$v_l = \tfrac{1}{2} v_g. \tag{31}$$

That the liquid velocity is proportional to the gas velocity is probably the most logical outcome for a flowing column of foam, but it is not inherently true of all foam columns. For instance, if more than half the bubbles burst at the top surface and the rate of bursting depends on the gas velocity, then the liquid and gas velocities will not be proportional to one another. This, in turn, will mean that the simple squared power law between liquid flowrate and gas velocity will not hold in that case.

It is interesting to note that, near the asymptote, the relationship between the velocity of the liquid (actual, rather than superficial) and that of the gas does not depend on the physical properties of liquid, such as viscosity and density. The liquid fraction and flowrate, though, do depend very strongly on the physical properties of the fluid.

5.2. Foam height

The second variable investigated experimentally was the effect of foam height on water recovery. Figure 3 shows that the full numerical solution predicts that an asymptotic water recovery will be approached very rapidly as foam height is increased. We were unable to investigate very shallow foams, but over a very wide range of greater foam heights no appreciable change in the water recovery is observed, which is exactly in line with the model's predictions. In figure 8, the gas rate was maintained at a value of 50 ml min^{-1}, while the foam height was varied.

6. Conclusions

This paper has produced a simple equation for obtaining the amount of liquid that flows out of a stable flowing foam in a column. This equation is the value of the liquid recovery that is rapidly approached as the foam height increases. For these deeper foams, the way in which the foam flows over the top of the column does not impact the result. For shallow froths, the flow profile over the lip does impact the water recovery and a set of one-dimensional equations were produced to solve the problem in this situation.

The lack of dependence of water recovery on foam height, above a certain height, was experimentally verified. Further, the predicted power law exponent between the water rate and gas rate of two was experimentally found.

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